Landau Damping in a Mixture of Bose and Fermi Superfluids

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We study the Landau damping in Bose-Fermi superfluid mixture at finite temperature. We find that at low temperature, the Landau damping rate will be exponentially suppressed at both the BCS side and the BEC side of Fermi superfluid. The momentum dependence of the damping rate is obtained, and it is quite different from the BCS side to the BEC side. The relations between our result and collective mode experiment in the recently realized Bose-Fermi superfluid mixture are also discussed.

I. INTRODUCTION

The quasiparticle is an important concept in modern many-body physics. The statistics and dispersion of quasiparticles determine various low energy properties of the quantum many-body system. In the general situation, there exsits interaction between the quasiparticles. That leads two major effects. One is the modification of the dispersion of excitations, corresponding to the real part of the self-energy of quasiparticle. Second, the interaction can also damp a given excitation, i.e. giving quasiparticle a finite lifetime, which can be reflected by the imaginary part of the self-energy. The damping of low energy excitations is responsible for many interesting phenomena of many-body system, such as transport and thermalization.

For example, in a uniform condensed Bose gas with short-range interaction, the low energy excitation is phonon-like quasiparticle with linear dispersion when wavelength is larger than the healing length. While when wavelength is smaller than healing length, the excitation is free-particle-like quasiparticle with quadratic dispersion. At zero temperature, due to the residual interaction between excitations, a quasiparticle in the BEC can be damped by decaying into two quasiparticles. Since each product of decay must have an energy lower than the original quasiparticle, the final state phase space is restricted. This gives the damping rate a very sensitive dependence on the initial momentum $\gamma \sim k^5$ [1]. This decay process is the so-called Beliaev damping. At finite temperature, a given quasiparticle can also be damped by absorbing thermal quasiparticles. As a result, it is very sensitive to temperature, $\gamma \sim T^4 k$ [2]. This mechanism is known as Landau damping, which was first discussed in plasma oscillation by Landau [3]. It plays an important role in various phenomena such as anomalous skin effect in metals and the damping of phonons in solids. Landau-Beliaev damping in a uniform Bose superfluid has been widely studied both theoretically [4][5][6][7] and experimentally [8]. In trapped system, since the low energy excitations are discrete, Beliaev damping is forbidden. The damping of low energy modes is attributed to Landau process, and the experiment damping rate has been found to be consistent with the theory of Landau damping [9][10]. Landau-Beliaev damping has also been studied in dipolar BECs [11] and in the mixture of BEC with normal Fermi gas [12].

Recently a Bose-Fermi superfluid mixture has been first realized by ENS group [13]. The dipole mode of this new superfluid mixture has been measured. It exhibits a frequency shift and an unusual damping behavior. This experimental development triggers many investigations on Bose-Fermi superfluid mixture [14][15][16][17][18][19].

The quasiparticles in this Bose-Fermi superfluid mixture have a quite unique feature. There are two gapless bosonic modes, which are Goldstone modes of Bose and Fermi superfluids. It also has a gapped fermionic mode, corresponding to the Cooper pair breaking. Moreover, in the ENS experiment, the Fermi superfluid can be tuned from the BCS side to the BEC side by Feshbach resonance. During the crossover, the behavior of three kinds of excitations gradually changes from the BCS limit to the BEC limit. In the BCS limit, the velocity of Goldstone mode in Fermi superfluid is quite large, approaching $v_F/\sqrt{3}$ [20], while the gap of the fermionic mode is exponentially small. When it is tuned to the BEC side, the velocity of the Goldstone mode decreases monotonously [21][22], while the gap becomes larger and larger. The dispersions of the three excitations at both sides are plotted in Fig.1.

To understand the unusual damping behavior of dipole mode in the ENS experiment, Zheng and Zhai study the Beliaev damping of bosonic mode in Bose superfluid by considering its interacting with quasiparticles in Fermi superfluid at zero temperature [17]. They found that Beliaev process will be activated only if the excitation momentum exceeding a critical value k_c. This threshold damping behavior, i.e. $\gamma \sim (k - k_c)^\alpha$, is quite different at the BCS side and the BEC side. To be specific, at the BCS side $\alpha = 0$, while at the BEC side $\alpha = 3$. This is because at the BCS side, the damping is dominated by decaying into fermionic pair-breaking modes in Fermi superfluid. The final state phase space of fermionic mode is restricted by density-of-state near the Fermi surface, so that the damping rate is nearly a constant. On the other hand, such a restriction does not exist at the BEC side,

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where the damping is dominated by decaying into Goldstone modes in Fermi superfluid. So the damping rate grows rapidly with momentum. However, they only consider the zero-temperature case, in which Landau damping is frozen. It is nature to ask the question, what will happen at finite temperature when Landau damping is activated.

In this paper, we investigate Landau damping of bosonic quasiparticles in Bose superfluid due to its interacting with quasiparticles in Fermi superfluid at finite temperature. We consider the typical cold atom situation, in which Fermi superfluid is in the strongly interacting regime, while Bose superfluid is in the weakly interacting regime. We find that unlike Beliaev damping, the critical momentum for Landau damping is zero, i.e. $k_c = 0$, both at the BCS side and the BEC side. We obtained the temperature dependence of Landau damping, showing that the damping rate is exponentially suppressed at temperature low enough at both sides. This is indeed a departure from the T^4 dependence in the single component BEC. The momentum dependence of the damping rate is also obtained, and its behavior is quite different from the BCS side to the BEC side. At BCS side, the damping rate grows linearly with momentum, while at the BEC side, it is nearly a constant. This damping behavior revealing different dominated low energy quasiparticles at each side.

This paper is organized as follows: In Sec. II we construct the model for Bose-Fermi superfluid mixture and its mean field treatment. In Sec. III Landau damping is considered by perturbation method at the BCS side. In Sec. IV Landau damping is calculated at the BEC side. In Sec. V, our main results are summarized and the connection to experiment is discussed.

II. THE MODEL

Consider a homogeneous mixture of Bose and Fermi superfluid. The Hamiltonian of the superfluid mixture has three parts: $\hat{H} = \hat{H}_b + \hat{H}_f + \hat{H}_{bf}$,

$$\begin{aligned}
\hat{H}_b &= \int d^3\mathbf{r} \left\{ \hat{b}^\dagger(\mathbf{r}) \hat{H}_{0,b} \hat{b}(\mathbf{r}) + \frac{g_b}{2} \hat{b}^\dagger(\mathbf{r}) \hat{b}^\dagger(\mathbf{r}) \hat{b}(\mathbf{r}) \hat{b}(\mathbf{r}) \right\}, \\
\hat{H}_f &= \int d^3\mathbf{r} \left\{ \sum_\sigma \hat{c}^\dagger_\sigma(\mathbf{r}) \hat{H}_{0,f} \hat{c}_\sigma(\mathbf{r}) \right. \\
&\quad \left. + g_f \hat{c}^\dagger_\uparrow(\mathbf{r}) \hat{c}^\dagger_\downarrow(\mathbf{r}) \hat{c}_\downarrow(\mathbf{r}) \hat{c}_\uparrow(\mathbf{r}) \right\}, \\
\hat{H}_{bf} &= g_{bf} \sum_\sigma \int d^3\mathbf{r} \hat{b}^\dagger(\mathbf{r}) \hat{b}(\mathbf{r}) \hat{c}^\dagger_\sigma(\mathbf{r}) \hat{c}_\sigma(\mathbf{r}),
\end{aligned} \tag{1}$$

where $\hat{H}_{0,i} = -\hbar^2\nabla^2/(2m_i) - \mu_i$, $i = b, f$ denotes bosons and fermions. $\sigma = \uparrow, \downarrow$ denotes the spin components of fermions. The coupling constants are related to the scattering lengths: $1/g_b = m_b/(4\pi\hbar^2 a_b)$ and $1/g_f = m_f/(4\pi\hbar^2 a_f) + \sum_\mathbf{k} 1/(2\varepsilon^f_\mathbf{k})$, where the scattering lengths can be tuned by Feshbach resonance. When a magnetic field is near a Feshbach resonance between fermions, g_b and g_{bf} are generally in the weakly interacting regime and are approximately constant according to the experiment setup. Therefore for $\hat{H}_b$, we take the Bogoliubov mean-field theory. For $\hat{H}_f$, we take the BCS-BEC crossover mean-field theory. After mean-field treatment, one obtains

$$\hat{H}^{\mathrm{MF}}_b = \sum_\mathbf{k} E^b_\mathbf{k} \hat{\alpha}^\dagger_\mathbf{k} \hat{\alpha}_\mathbf{k}, \tag{2}$$

$$\hat{H}^{\mathrm{MF}}_f = \sum_\mathbf{k} E^f_\mathbf{k} (\hat{\beta}^\dagger_\mathbf{k} \hat{\beta}_\mathbf{k} + \hat{\gamma}^\dagger_\mathbf{k} \hat{\gamma}_\mathbf{k}), \tag{3}$$

where $E^b_\mathbf{k} = \sqrt{\varepsilon^b_\mathbf{k} \left(\varepsilon^b_\mathbf{k} + 2g_b n_b \right)}$, $\hat{\alpha}_k$ is the quasiparticle operator for bosonic Goldstone mode in Bose superfluid. $E^f_\mathbf{k} = \sqrt{(\varepsilon^f_\mathbf{k} - \mu_f)^2 + \Delta^2}$. $\hat{\beta}_\mathbf{k}$ and $\hat{\gamma}_\mathbf{k}$ are the quasiparticle operators for fermionic pair-breaking mode in Fermi superfluid. $\varepsilon^i_\mathbf{k} = \hbar^2 k^2/(2m_i)$, $i = b, f$ denotes the kinetic energy of bosons and fermions. The corresponding Bogoliubov transformations of these operators are given by

$$\hat{b}_\mathbf{k} = u^b_\mathbf{k} \hat{\alpha}_\mathbf{k} - v^b_\mathbf{k} \hat{\alpha}^\dagger_{-\mathbf{k}}, \tag{4}$$

$$\hat{c}_{\mathbf{k},\uparrow} = u^f_\mathbf{k} \hat{\beta}_\mathbf{k} + v^f_\mathbf{k} \hat{\gamma}^\dagger_{-\mathbf{k}}, \tag{5}$$

$$\hat{c}_{\mathbf{k},\downarrow} = u^f_\mathbf{k} \hat{\gamma}_\mathbf{k} - v^f_\mathbf{k} \hat{\beta}^\dagger_{-\mathbf{k}}. \tag{6}$$

Here momentum-dependent coefficients $u_\mathbf{k}$ and $v_\mathbf{k}$ are given by

$$u^b_\mathbf{k}(v^b_\mathbf{k}) = \sqrt{\frac{1}{2} \left(\frac{\varepsilon^b_\mathbf{k} + g_b n_b}{E^b_\mathbf{k}} \pm 1 \right)}, \tag{7}$$

$$u^f_\mathbf{k}(v^f_\mathbf{k}) = \sqrt{\frac{1}{2} \left(1 \pm \frac{\varepsilon^f_\mathbf{k} - \mu_f}{E^f_\mathbf{k}} \right)}. \tag{8}$$

For fermionic pair-breaking mode in Fermi superfluid, when $-1/k_F a_f$ gets smaller from the BCS side to the BEC side, Δ will increase and μ_f will decrease. Apart from the pair-breaking mode, there is also a bosonic mode of the center-of-mass motion of Cooper pairs in Fermi superfluid, which is beyond BCS-BEC mean-field theory. Its dispersion relation at low energy is linear: $E^m_\mathbf{k} = \hbar c_f k$. From the BCS side to the BEC side, c_f evolves from $v_F/\sqrt{3}$ to $\sqrt{\pi\hbar^2 a_m n_m / m_f^2}$ [21][22], where $a_m = 0.6 a_f$ [23], and $n_m = n_\uparrow = n_\downarrow = n_f$.

Therefore there are three different excitations in Bose-Fermi superfluid mixture: the bosonic Goldstone mode in Bose superfluid, whose dispersion is given by $E^b_\mathbf{k}$, the fermionic pair-breaking mode in Fermi superfluid, whose dispersion is given by $E^f_\mathbf{k}$ and the bosonic Goldstone mode in Fermi superfluid, whose dispersion is given by $E^m_\mathbf{k}$. At the BCS side, the fermions form Cooper pairs and become superfluid of BCS type. At the BEC side, the fermions form strongly bound molecules and become superfluid of BEC type. The dispersions of the three

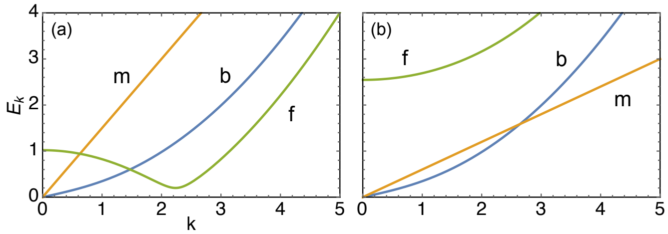

FIG. 1: Schematic of dispersions of the bosonic Goldstone mode in Bose superfluid ($E^b_{\mathbf{k}}$), the fermionic pair-breaking mode in Fermi superfluid($E^f_{\mathbf{k}}$) and the bosonic Goldstone mode in Fermi superfluid ($E^m_{\mathbf{k}}$). (a) is in the BCS side and (b) is in the BEC side.

excitations at different sides are shown respectively in Fig. 1(a) and (b). In the experiment setup, the Bose gas is so dilute that we can take the free-particle limit, i.e. $E^b_{\mathbf{k}} \approx \varepsilon^b_{\mathbf{k}}$ for the Bogoliubov mode in the Bose superfluid. In this case, Landau-Beliaev damping of the bosonic mode in Bose superfluid due to its interaction with itself can be ignored.

III. DAMPING AT THE BCS SIDE

Consider the interaction between bosons and fermions. Due to the existence of boson condensate, $\hat{b}_0$ and $\hat{b}^\dagger_0$ can be treated as c-numbers, i.e. $\hat{b}_0 = \hat{b}^\dagger_0 = \sqrt{N_b}$. Expand the Hamiltonian by the order of $\sqrt{N_b}$, we have $\hat{H}_{bf} = \hat{H}^{(1)}_{bf} + \hat{H}^{(2)}_{bf} + \hat{H}^{(3)}_{bf}$, where

$$\hat{H}^{(1)}_{bf} = g_{bf} n_b \sum_{\mathbf{k},\sigma} \hat{c}^\dagger_{\mathbf{k},\sigma} \hat{c}_{\mathbf{k},\sigma}, \tag{9}$$

$$\hat{H}^{(2)}_{bf} = \frac{g_{bf}\sqrt{N_b}}{V} \sum_{\mathbf{k}\neq 0,\mathbf{q},\sigma} (\hat{c}^\dagger_{\mathbf{q+k},\sigma} \hat{c}_{\mathbf{q},\sigma} \hat{b}_{\mathbf{k}} + \text{h.c.}), \tag{10}$$

$$\hat{H}^{(3)}_{bf} = \frac{g_{bf}}{V} \sum_{\mathbf{k},\mathbf{q}\neq 0,\mathbf{p},\sigma} \hat{c}^\dagger_{\mathbf{p-q},\sigma} \hat{c}_{\mathbf{p-k},\sigma} \hat{b}^\dagger_{\mathbf{q}} \hat{b}_{\mathbf{k}}. \tag{11}$$

The leading term $\hat{H}^{(1)}_{bf}$ will only shift the chemical potential of fermions. The subleading term $\hat{H}^{(2)}_{bf}$ will induce the damping of boson quasiparticles. The last term $\hat{H}^{(3)}_{bf}$ is a two particle scattering process, which is less important compared to $\hat{H}^{(2)}_{bf}$, so that it can be ignored. We get several damping channels by expressing $\hat{H}^{(2)}_{bf}$ with mean-field quasiparticle operators: $\hat{H}^{(2)}_{bf} \approx \hat{H}_1 + \hat{H}_2 + \hat{H}_3$. Here

$$\hat{H}_1 = \frac{g_{bf}\sqrt{N_b}}{V} \sum_{\mathbf{k},\mathbf{q},\sigma} (u^b_{\mathbf{k}} - v^b_{\mathbf{k}})(u^f_{\mathbf{q+k}} u^f_{\mathbf{q}} - v^f_{\mathbf{q+k}} v^f_{\mathbf{q}}) \hat{\beta}^\dagger_{\mathbf{q+k}} \hat{\beta}_{\mathbf{q}} \hat{\alpha}_{\mathbf{k}} + \text{h.c.} \tag{12}$$

$$\hat{H}_2 = \frac{g_{bf}\sqrt{N_b}}{V} \sum_{\mathbf{k},\mathbf{q},\sigma} (u^b_{\mathbf{k}} - v^b_{\mathbf{k}})(u^f_{\mathbf{q+k}} u^f_{\mathbf{q}} - v^f_{\mathbf{q+k}} v^f_{\mathbf{q}}) \hat{\gamma}^\dagger_{\mathbf{q+k}} \hat{\gamma}_{\mathbf{q}} \hat{\alpha}_{\mathbf{k}} + \text{h.c.} \tag{13}$$

$$\hat{H}_3 = \frac{g_{bf}\sqrt{N_b}}{V} \sum_{\mathbf{k},\mathbf{q},\sigma} (u^b_{\mathbf{k}} - v^b_{\mathbf{k}})(u^f_{\mathbf{k-q}} v^f_{\mathbf{q}} - v^f_{\mathbf{k-q}} u^f_{\mathbf{q}}) \hat{\gamma}^\dagger_{\mathbf{q}} \hat{\beta}^\dagger_{\mathbf{k-q}} \hat{\alpha}_{\mathbf{k}} + \text{h.c.} \tag{14}$$

We have ignored terms like $\hat{\beta}_{\mathbf{q}} \hat{\beta}_{\mathbf{q-k}} \hat{\alpha}_{\mathbf{k}}$ that violate the conservation of energy, which will not contribute to damping process.

We note that $\hat{H}_3$ is the decay process of bosonic mode in Bose superfluid, which is Beliaev damping. $\hat{H}_1$ and $\hat{H}_2$ are the scattering of the bosonic mode in Bose superfluid by thermal excitations, corresponding to Landau damping. Unlike Landau-Beliaev damping in single component Bose gas, there exists nonzero damping threshold for some of the damping channels. While due to the energy gap Δ in $\hat{\beta}_{\mathbf{q+k}}$ and $\hat{\gamma}_{\mathbf{q+k}}$. The damping threshold for $\hat{H}_3$ is $\Omega_3(\mathbf{k}) = \min[E^f_{\mathbf{q}} + E^f_{\mathbf{k-q}}] > 2\Delta$ which is nonzero. That is to say, for low energy excitations with $E^b_{\mathbf{k}} < \Omega_3(\mathbf{k})$, there will be no damping contributed from channel $\hat{H}_3$. So the Beliaev damping has a critical momentum. The damping threshold of $\hat{H}_1$ and $\hat{H}_2$ are the same, which is $\Omega_1(\mathbf{k}) = \Omega_2(\mathbf{k}) = \min[E^f_{\mathbf{q+k}} - E^f_{\mathbf{q}}] = 0$. So the Landau damping we considered here has no damping threshold, i.e. the critical momentum for the damping is zero.

As shown in Fig. 1(a), there is also a phonon-like bosonic mode of center-of-mass motion of Cooper pairs in Fermi superfluid. We will discuss its contribution to the Landau damping later.

Since the interaction between bosons and fermions is weak compared to the excitation energy in both bosons and fermions, so that we can treat it perturbatively. According to Fermi's Golden Rule, the rate for the Landau

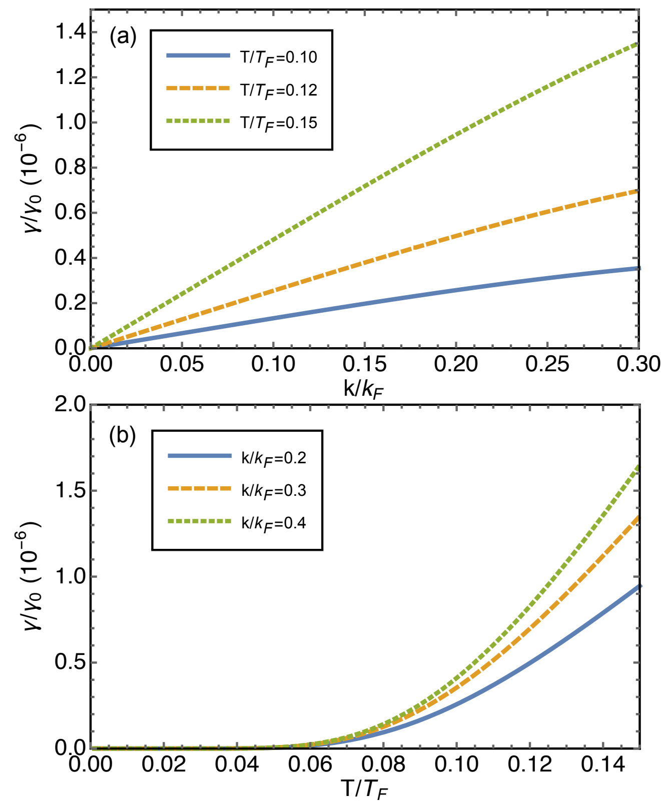

FIG. 2: Damping rates γ in units of γ_0 ($\gamma_0 \equiv E_F/\hbar$) as a function of k/k_F with $1/(k_F a_f) = -0.5$ in the BCS side. (a) Damping rates as a function of momentum; (b) Damping rates as a function of temperature.

damping described by $\hat{H}_1$ and $\hat{H}_2$ is given by

$$\gamma(\mathbf{k}) = \frac{2\pi}{\hbar}\sum_{\mathbf{q}}|M_{\mathbf{qk}}|^2\delta(E^b_{\mathbf{k}}+E^f_{\mathbf{q}}-E^f_{\mathbf{q+k}})\left[f(E^f_{\mathbf{q}}) - f(E^f_{\mathbf{q+k}})\right], \tag{15}$$

where $f(E^f_{\mathbf{q}}) = [\exp(E^f_{\mathbf{q}}/T) + 1]^{-1}$ is the Fermi-Dirac distribution function. The matrix element is

$$M_{\mathbf{qk}} = \frac{2g_{bf}\sqrt{N_b}}{V}(u^b_{\mathbf{k}} - v^b_{\mathbf{k}})(u^f_{\mathbf{q+k}}u^f_{\mathbf{q}} - v^f_{\mathbf{q+k}}v^f_{\mathbf{q}}). \tag{16}$$

The damping rate given by (15) is numerically calculated and the result is plotted in Fig. 2.

In Fig. 2(a), the damping rate is zero for $k = 0$. It is linear in k when k is small. The linear behavior is the result of matrix element and density-of-state of quasiparticles together. The slope is larger at higher temperature due to more thermal excitations.

What is more interesting is the temperature dependence of Landau damping rate. As shown in Fig. 2(b), when $T/T_F \ll 1$, the damping rate shows a $e^{-T_F/T}$ behavior. This exponential decay is very different from the power law behavior of Landau damping in dilute Bose gas. Similar as the damping threshold discussed before, the conservation of energy must be satisfied in the damping process. Due to the existence of the gap Δ in $\hat{\beta}_{\mathbf{k}}$ and $\hat{\gamma}_{\mathbf{k}}$, in order for quasiparticle to be damped, it must absorb a thermal excitation with $E^f_{\mathbf{q}} \gtrsim \Delta$. However, as temperature tends to zero, the distribution function $f(E^f_{\mathbf{q}})$ tends to be $e^{-\Delta/(k_B T)}$. The number of thermal excitations is exponentially suppressed at low temperature $k_B T < \Delta$. Therefore the damping rate is also exponentially suppressed. The size of the suppressed region is proportional to gap. The region will be larger if the BCS gap is tuned to be larger. For usual Landau damping in dilute Bose gas, thermal excitations are gapless. So even at low temperature, a number of thermal excitations can be excited and contribute to damping, leading to a T^4 power law temperature dependence.

IV. DAMPING AT THE BEC SIDE

According to the result at the BCS side, the damping rate contributed by the pair-breaking channel will have a exponentially suppressed region. In the BEC side the molecule is tightly bounded and the pair-breaking energy Δ is quite high. So this suppressed region is large. This damping channel can be neglected.

Now the damping channel of center-of-mass motion of Cooper pairs in Fermi superfluid is important. A comprehensive description of Goldstone mode in Fermi superfluid and it coupling to Bose superfluid can be obtained from fluctuation theory of Fermi superfluid [24]. Here to expose physics in a simple way, we treat Fermi superfluid at the BEC side as a molecular BEC. So the Hamiltonian is given by three parts: $\hat{H} = \hat{H}_b + \hat{H}_m + \hat{H}_{bm}$. H_b is the same as before.

$$\hat{H}_m = \int d^3\mathbf{r}\left\{\hat{d}^\dagger(\mathbf{r})\hat{H}_{0,m}\hat{d}(\mathbf{r}) + \frac{g_m}{2}\hat{d}^\dagger(\mathbf{r})\hat{d}^\dagger(\mathbf{r})\hat{d}(\mathbf{r})\hat{d}(\mathbf{r})\right\},$$
$$\hat{H}_{bm} = g_{bm}\int d^3\mathbf{r}\hat{d}^\dagger(\mathbf{r})\hat{d}(\mathbf{r})\hat{b}^\dagger(\mathbf{r})\hat{b}(\mathbf{r}), \tag{17}$$

where $E_{0,m} = -\hbar^2\nabla^2/(2m_m) - \mu_m$. $1/g_m = m_m/(4\pi\hbar^2 a_m)$. $m_m = 2m_f$ and $a_m = 0.6a_f$. For $\hat{H}_m$, we take the Bogoliubov mean-field theory. The result is

$$\hat{H}^{\rm MF}_m = \sum_{\mathbf{k}} E^m_{\mathbf{k}}\hat{\chi}^\dagger_{\mathbf{k}}\hat{\chi}_{\mathbf{k}}, \tag{18}$$

where $E^m_{\mathbf{k}} = \sqrt{\varepsilon^m_{\mathbf{k}}(\varepsilon^m_{\mathbf{k}} + 2g_m n_m)}$, $\hat{\chi}_k$ is the quasiparticle operator for bosonic Goldstone mode in molecular BEC. $\varepsilon^m_{\mathbf{k}} = \hbar^2k^2/(2m_m)$ is the kinetic energy of molecules. The corresponding Bogoliubov transformation of molecule operators is given by $d_{\mathbf{k}} = u^b_{\mathbf{k}}\hat{\chi}_{\mathbf{k}} - v^b_{\mathbf{k}}\hat{\chi}^\dagger_{-\mathbf{k}}$, where momentum-dependent coefficients $u^m_{\mathbf{k}}$ and $v^m_{\mathbf{k}}$ are given by

$$u^m_{\mathbf{k}}(v^m_{\mathbf{k}}) = \sqrt{\frac{1}{2}\left(\frac{\varepsilon^m_{\mathbf{k}} + g_m n_m}{E^m_{\mathbf{k}}} \pm 1\right)}. \tag{19}$$

Due to the existence of condensates, $\hat{b}_0$, $\hat{b}^\dagger_0$, $\hat{d}_0$ and $\hat{d}^\dagger_0$ can be replaced by $\hat{b}_0 = \hat{b}^\dagger_0 = \sqrt{N_b}$, $\hat{d}_0 = \hat{d}^\dagger_0 = \sqrt{N_m}$. To

the order of $\sqrt{N_b}$ and $\sqrt{N_m}$, $\hat{H}_{bm}=\hat{H}^{(1)}_{bm}+\hat{H}^{(2)}_{bm}+\hat{H}^{(3)}_{bm}$, where

$$\hat{H}^{(1)}_{bm} = g_{bm}\sum_{\mathbf{k}\neq 0}(n_m\hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}+n_b\hat{d}^\dagger_{\mathbf{k}}\hat{d}_{\mathbf{k}}), \tag{20}$$

$$\begin{aligned}\hat{H}^{(2)}_{bm} =& \frac{g_{bm}\sqrt{N_m}}{V}\sum_{\mathbf{k},\mathbf{q}\neq 0}(\hat{d}_{\mathbf{q}}+\hat{d}^\dagger_{-\mathbf{q}})\hat{b}^\dagger_{\mathbf{k}+\mathbf{q}}\hat{b}_{\mathbf{k}} \\ &+\frac{g_{bm}\sqrt{N_b}}{V}\sum_{\mathbf{k},\mathbf{q}\neq 0}(\hat{d}^\dagger_{\mathbf{q}+\mathbf{k}}\hat{d}_{\mathbf{q}}\hat{b}_{\mathbf{k}}+\hat{d}^\dagger_{\mathbf{q}-\mathbf{k}}\hat{d}_{\mathbf{q}}\hat{b}^\dagger_{\mathbf{k}}),\end{aligned} \tag{21}$$

$$\hat{H}^{(3)}_{bm} = \frac{g_{bm}}{V}\sum_{\mathbf{p},\mathbf{q}\neq 0,\mathbf{k}\neq\mathbf{q}}\hat{d}^\dagger_{\mathbf{q}-\mathbf{k}}\hat{d}_{\mathbf{q}}\hat{b}^\dagger_{\mathbf{p}+\mathbf{k}}\hat{b}_{\mathbf{p}}. \tag{22}$$

The leading term $\hat{H}^{(1)}_{bm}$ will only modify the chemical potential. The subleading term $\hat{H}^{(2)}_{bm}$ will contribute to the damping. The last term $\hat{H}^{(3)}_{bf}$ is a two particle scattering process, which is less important compared to the $\hat{H}^{(2)}_{bf}$ and can be ignored. Express $\hat{H}^{(2)}_{bm}$ with mean-field quasiparticle operators, we get damping channels $\hat{H}^{(2)}_{bm}\approx\hat{H}_1+\hat{H}_2+\hat{H}_3+\hat{H}_4$, where

$$\hat{H}_1 = \frac{g_{bm}\sqrt{N_m}}{V}\sum_{\mathbf{k},\mathbf{q}}(u^m_{\mathbf{q}}-v^m_{\mathbf{q}})(u^b_{\mathbf{k}-\mathbf{q}}u^b_{\mathbf{k}}+v^b_{\mathbf{k}-\mathbf{q}}v^b_{\mathbf{k}})\hat{\chi}^\dagger_{\mathbf{q}}\hat{\alpha}^\dagger_{\mathbf{k}-\mathbf{q}}\hat{\alpha}_{\mathbf{k}}+\text{h.c.}, \tag{23}$$

$$\hat{H}_2 = -\frac{g_{bm}\sqrt{N_b}}{V}\sum_{\mathbf{k},\mathbf{q}}(u^b_{\mathbf{k}}-v^b_{\mathbf{k}})u^m_{\mathbf{k}-\mathbf{q}}v^m_{\mathbf{q}}\hat{\chi}^\dagger_{\mathbf{q}}\hat{\chi}^\dagger_{\mathbf{k}-\mathbf{q}}\hat{\alpha}_{\mathbf{k}}+\text{h.c.}, \tag{24}$$

$$\hat{H}_3 = -\frac{g_{bm}\sqrt{N_m}}{V}\sum_{\mathbf{k},\mathbf{q}}(u^m_{\mathbf{q}+\mathbf{k}}-v^m_{\mathbf{q}+\mathbf{k}})u^b_{\mathbf{k}}v^b_{\mathbf{q}}\hat{\chi}^\dagger_{\mathbf{q}+\mathbf{k}}\hat{\alpha}_{\mathbf{q}}\hat{\alpha}_{\mathbf{k}}+\text{h.c.}, \tag{25}$$

$$\hat{H}_4 = \frac{g_{bm}\sqrt{N_b}}{V}\sum_{\mathbf{k},\mathbf{q}}(u^b_{\mathbf{k}}-v^b_{\mathbf{k}})(u^m_{\mathbf{q}+\mathbf{k}}u^m_{\mathbf{q}}+v^m_{\mathbf{q}+\mathbf{k}}v^m_{\mathbf{q}})\hat{\chi}^\dagger_{\mathbf{q}+\mathbf{k}}\hat{\chi}_{\mathbf{q}}\hat{\alpha}_{\mathbf{k}}+\text{h.c.}, \tag{26}$$

We have ignored terms like $\hat{\chi}_{-\mathbf{q}}\hat{\alpha}_{\mathbf{q}+\mathbf{k}}\hat{\alpha}_{\mathbf{k}}$ that violates the conservation of energy that will not contribute to damping process.

Here $\hat{H}_2$ is the decay of bosonic mode in Bose superfluid, which is Beliaev damping. $\hat{H}_3$ and $\hat{H}_4$ are scattering by thermal excitations, which are Landau damping. Special attention should be paid on $\hat{H}_1$: $\hat{\chi}^\dagger_{\mathbf{q}}\hat{\alpha}^\dagger_{\mathbf{k}-\mathbf{q}}\hat{\alpha}_{\mathbf{k}}$ gives Beliaev damping, while its Hermitian conjugation $\hat{\alpha}^\dagger_{\mathbf{q}+\mathbf{k}}\hat{\chi}_{\mathbf{q}}\hat{\alpha}_{\mathbf{k}}$ can contribute to Landau damping. Similar as the BCS side, $\hat{\alpha}_{\mathbf{k}}$ and $\hat{\chi}_{\mathbf{k}}$ are different quasiparticles with different dispersions. To satisfy conservation of energy and momentum, the Beliaev damping channels $\hat{H}_1$ and $\hat{H}_2$ have damping thresholds for low-energy excitations:

$$\begin{aligned}\Omega_1(\mathbf{k}) &= \begin{cases}k^2/(2m_b) & k<m_bc_m,\\ c_mk-m_bc_m^2/2 & k>m_bc_m,\end{cases}\\ \Omega_2(\mathbf{k}) &= c_mk,\end{aligned} \tag{27}$$

respectively. The Landau damping channels have no damping threshold, i.e. have a zero critical momentum.

The matrix elements of Landau damping are given by:

$$\begin{aligned}M_{1,\mathbf{qk}} &= \frac{g_{bm}\sqrt{N_m}}{V}(u^m_{\mathbf{q}}-v^m_{\mathbf{q}})(u^b_{\mathbf{k}-\mathbf{q}}u^b_{\mathbf{k}}+v^b_{\mathbf{k}-\mathbf{q}}v^b_{\mathbf{k}}),\\ M_{3,\mathbf{qk}} &= -\frac{g_{bm}\sqrt{N_m}}{V}(u^m_{\mathbf{q}+\mathbf{k}}-v^m_{\mathbf{q}+\mathbf{k}})(u^b_{\mathbf{k}}v^b_{\mathbf{q}}+v^b_{\mathbf{k}}u^b_{\mathbf{q}}),\\ M_{4,\mathbf{qk}} &= \frac{g_{bm}\sqrt{N_b}}{V}(u^b_{\mathbf{k}}-v^b_{\mathbf{k}})(u^m_{\mathbf{q}+\mathbf{k}}u^m_{\mathbf{q}}+v^m_{\mathbf{q}+\mathbf{k}}v^m_{\mathbf{q}}).\end{aligned} \tag{28}$$

As mentioned before, we only consider the free-particle-limit for Goldstone mode in Bose superfluid. So $u^b_{\mathbf{k}}$ and $v^b_{\mathbf{k}}$ can be simplified as $u^b_{\mathbf{k}}\approx 1$, $v^b_{\mathbf{k}}\approx 0$. We immediately get that $M_{1,\mathbf{qk}}=M_{3,\mathbf{qk}}=0$. Physically, this is because in the free-boson limit $\hat{\alpha}_{\mathbf{k}}\approx\hat{b}_{\mathbf{k}}$. Thus $\hat{H}_1$ and $\hat{H}_3$ violate the conservation of number of bosons. Therefore we only consider damping channel $\hat{H}_4$.

According to Fermi's Golden Rule, this Landau damping rate is given by

$$\gamma(\mathbf{k}) = \frac{2\pi}{\hbar}\sum_{\mathbf{q}}|M_{\mathbf{qk}}|^2\delta(E^b_{\mathbf{k}}+E^m_{\mathbf{q}-\mathbf{k}}-E^m_{\mathbf{q}})\left[f(E^m_{\mathbf{q}-\mathbf{k}})-f(E^m_{\mathbf{q}})\right], \tag{29}$$

where $f(E^m_{\mathbf{q}})=[\exp(E^m_{\mathbf{q}}/T)-1]^{-1}$ is the Bose-Einstein distribution function. The conservation of energy $E^b_{\mathbf{k}}+E^m_{\mathbf{q}-\mathbf{k}}-E^m_{\mathbf{q}}=0$ will give a momentum lower bound $q_c(\mathbf{k})$

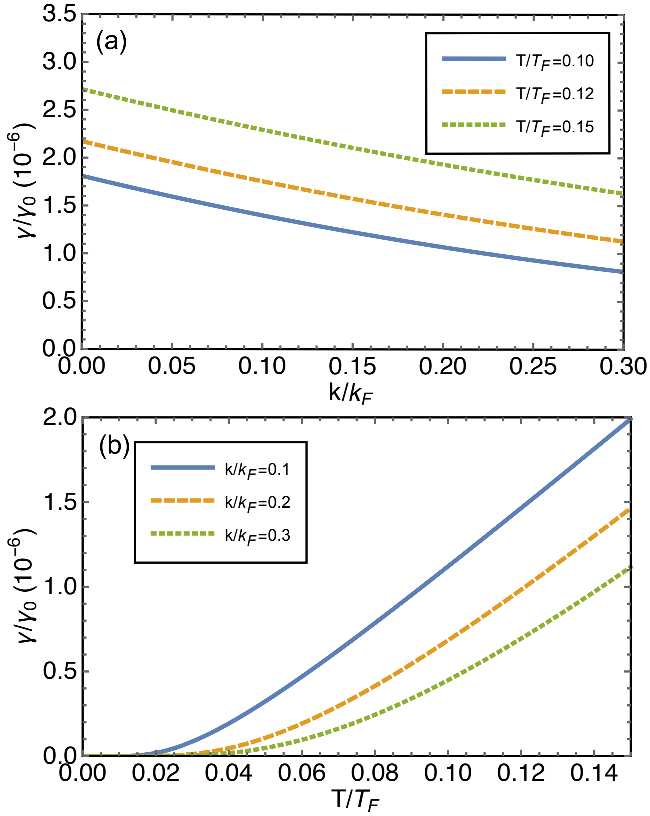

FIG. 3: Damping rates γ in units of γ_0 ($\gamma_0 \equiv E_F/\hbar$) as a function of k/k_F with $1/(k_F a_f) = 0.5$ for the BEC side. (a) Damping rates as a function of momentum; (b) Damping rates as a function of temperature.

for the thermal excitations, which is

$$q_c(\mathbf{k}) = \frac{k}{2}\left(1 - \frac{\hbar k}{2m_b c_m}\right). \tag{30}$$

Only thermal excitations with $q > q_c(\mathbf{k})$ will contribute to the damping. The physical reason for this lower bound can be understood similar as the damping threshold as following: for infinitesimal $\mathbf{q}$, the conservation of energy and momentum cannot be satisfied simultaneously, because $\hat{\chi}_{\mathbf{k}}$ are quasiparticles with different dispersions from $\hat{\alpha}_{\mathbf{k}}$. To be more specific, $\hat{\chi}_{\mathbf{k}}$ has a large velocity and has higher energy compared to $\hat{\alpha}_{\mathbf{k}}$.

The damping rate given by (29) is numerically calculated and the result is plotted in Fig. 3. In Fig. 3(a), the damping rate is nonzero at $k = 0$. At leading order, the damping rate decreases with momentum. This can be understood as the competition of matrix element and the density-of-state of the quasiparticles. The leading and the subleading term of $\gamma(\mathbf{k})$ can be calculated analytically by taking the asymptotic expansion of matrix element. The result is

$$\gamma(\mathbf{k}) = \frac{g_{bm}^2 n_b m_m^2}{4\pi\hbar^3 m_b}\left[\frac{2m_m c_m T}{\hbar T_F} - \left(\frac{1}{2} + \frac{m_m T}{m_b T_F}\right)k\right] + \mathcal{O}(k^2). \tag{31}$$

The intercept at $k = 0$ is proportional to T/T_F. This is reasonable because when $T = 0$, there are no thermal excitations. The subleading term is always negative, which agrees with the numerical calculations, see Fig. 3(a).

The temperature dependence of the damping rate is plottd in Fig. 3(b). We note that $\gamma(T/T_F)$ is linear in T/T_F at high temperature. At low temperature, when $T/T_F \ll 1$, $\gamma(T/T_F) \propto e^{-T_F/T}$. So at the BEC side, the temperature dependence also shows an exponentially decay behavior, which is similar to the BCS side. However, the physical origin of this exponentially suppression is not the gap, but the momentum lower bound. Since only excitations with $q > q_c$ can contribute to the damping, there is an energy lower bound $E_c = \hbar c_m q_c$ for thermal excitations to involve into Landau damping process. For low temperature $k_B T < E_c$, the involved thermal excitations are exponentially suppressed, leading to an exponentially suppressed damping rate. The size of this suppressed region is proportional to energy lower bound E_c. According to (30), larger the k, larger the E_c, hence larger the suppressed region. This is agreed with Fig. 3(b) and is also a distinct feature from the BCS side. For the Landau damping in single component BEC, the quasiparticles are also gapless. But there is only one kind of quasiparticle and thus no energy lower bound. The conservation of energy and momentum can always be satisfied there. As a result, there is no exponential suppressed region.

Now we would like to point out that the Landau damping channels via interacting with the Goldstone mode in Fermi superfluid also exist at the BCS side. Although the detail of these channels cannot be covered at mean-field level, they must obey the conservation of energy and momentum. The velocity of Goldstone mode in Fermi superfluid at the BCS side is about $v_F/\sqrt{3}$ and is quite large. So based on our previous analysis, the corresponding energy lower bound $\hbar c_m q_c$ is also very large at the BCS side, such that the damping process via those channels are highly suppressed in a large temperature range. It can be ignored comparing to the damping channels via interacting with the fermionic pair breaking modes at the BCS side.

At the end of this section, we should emphasize that our analysis of temperature dependence in Landau damping is only viable at a relative low temperature, since both the gap of the Fermi superfluid and the condensate fraction in Bose superfluid will decrease with the increasing of temperature. A comprehensive self-consistent analysis of temperature dependence should take the changing of the gap and the condensate fraction into account. However, at low temperature both the gap and the condensate fraction are not sensitive to the temperature, so our result is still reasonable in this region.

V. SUMMARY

In summary, we have studied Landau damping in Bose-Fermi superfluid mixture at finite temperature. Unlike Beliaev damping in Bose-Fermi superfluid mixture at zero temperature, Landau damping has no critical momentum, since any quasiparticle with infinitesimal momentum can be damped by absorbing a thermal quasiparticle. However, due to energy-momentum conservation, thermal excitations in Fermi superfluid will be involved into Landau process only if their energy is larger than a lower bound E_c. When the temperature is lower than E_c, Landau damping rate will be exponentially suppressed. For fermionic quasiparticles in Fermi superfluid, the lower bound is determined by pair-breaking gap Δ. For bosonic excitations, the lower bound is determined by $\hbar c_m q_c$. This exponential suppression is quite different from the power law temperature dependence of Landau damping in single component BEC. The reason behind is that in Bose-Fermi superfluid mixture, quasiparticle in Bose superfluid is damped by coupling to thermal excitations in Fermi superfluids with totally different dispersion. In single component BEC, quasiparticle is damped by interacting with itself, so that energy-momentum conservation can always be satisfied. Therefore there will be no energy lower bound and no exponential suppression. In dipolar Bose gas, there is also an exponential suppression region for Landau damping [11]. That is due to its unusual quasiparticle dispersion with maxon-roton structure.

In principle, both damping channels contributed from bosonic quasiparticles and fermionic quasiparticles in Fermi superfluid exist at both sides. However, at the BCS side, we have $\Delta \ll \hbar c_m q_c$, so that Landau damping is dominated by fermionic excitations in Fermi superfluid. At the BEC side, the situation is opposite, $\Delta \gg \hbar c_m q_c$. So bosonic quasiparticles in Fermi superfluid dominate Landau process. This result is similar to the case of zero-temperature Beliaev damping. The different dominated low energy quasiparticles also lead to totally distinct momentum dependence of damping rate at the BCS side and the BEC side.

In the ENS experiment [13], damping of dipole modes has a critical momentum even the experiment is done at finite temperature. Based on our previous analysis, this phenomenon can be understood as following: the temperature of the experiment is sufficient low that Landau damping with zero critical momentum is highly suppressed. Then Beliaev damping with nonzero critical momentum will show up [17]. Our prediction of momentum dependence of Landau damping could be observed in the same experiment setup at relative high temperature, when Landau damping will dominate over Beliaev damping. No critical momentum will be observed there.

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